

AcerMC Monte Carlo generator and heavy flavor matching

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- massive PS+ME matching: A glimpse of the theory
- AcerMC: Some facts
- massive PS+ME matching in AcerMC: Examples of the method

#### Factorization theorem

Let us start with the basic recipe for calculating the cross-section for hadroproduction of <sup>a</sup> certain final state (involving QCD):

- Use your favorite method of dealing with Feynman diagrams to calculate the process of interest (quarks, <sup>g</sup>luons going into the process, resulting in final state X) @LO,@NLO, whatever perturbative level you can manage.
- Apply some phase space integration to get the parton-level (hard) cross-section  $\hat{\sigma}_{ab\rightarrow X}$ .
- In experiments one does not collide partons but hadrons, thus multiply the result with the probabilities to ge<sup>t</sup> partons of certain flavor i and energy fraction  $\xi$  from the parent hadron I at a certain energy scale  $\mu_F$ : parton distribution functions  $f_{i/I}(\xi_i,\mu_F)$ , fitted from experimental data, freely available (MRST, CTEQ...). Feel free to integrate again over left-over parameters.

Given as <sup>a</sup> formula, one would produce:

$$
\sigma_{AB\to X}=\sum_{a,b}f_{a/A}\otimes \hat{\sigma}_{ab\to X}\otimes f_{b/B}=\sum_{a,b}\int\frac{d\xi_a}{\xi_a}\int\frac{d\xi_b}{\xi_b}f_{a/A}(\xi_a,\mu_F)\,f_{b/B}(\xi_b,\mu_F)\,\hat{\sigma}_{ab\to X}(\xi_a,\xi_b,\mu_F\ldots),
$$

also commonly known as the Factorization Theorem.

- The incoming partons  $a, b$  are generally treated as massless.
- There is however a problem with this method, it's not really correct...

## Factorization theorem cont'd

The point is, one needs to take the Factorization Theorem seriously:

- The hard (parton-level) cross-section  $\hat{\sigma}_{ab\to X}$  really needs to be *hard* (short-distance, high-energy) in every respect.
- All the soft (long-distance, low-energy) effects are formally swallowed by the PDFs  $f_{i/I}(\xi_i,\mu_F)$ .
- The factorization scale  $\mu_F$  sets the dividing limit.
- The perturbative calculation (Feynman diagrams) is by its method not necessarily hard. While one can formally remove most of the possible divergencies in the perturbative calculation (IR vs UV cancellation etc) the long-distance effects show up as mass/collinear divergencies in form of logs  $\alpha_s \log(\mu_F^2/m^2)$ .



- Such cases must be *removed* from the hard cross-section and *moved* to the PDFs  $f_{i/I}(\xi_i,\mu_F)$ .
- This is also what the DGLAP equations tell you:

$$
\frac{d}{d\ln\mu_F^2}f_{i/I}(z,\mu_F)=\frac{\alpha_s(\mu_F)}{2\pi}\sum_j\int\limits_{z}^1\frac{d\xi}{\xi}P_{j\to i}(\frac{z}{\xi},\alpha_s(\mu_F))\,f_{j/I}(\xi,\mu_F).
$$

## Factorization theorem cont'd

So let's see how this would work in an example: Let us assume you have <sup>a</sup> gluon entering your perturbative calculation, which then splits to a quark pair  $g \to H\bar{H}$ . Stipulating, that the (hard/soft) scale  $\mu$  is set by the heavy quark propagator (alternatively,  $p_T$  of the spectator is possible):

- If the scale is hard enough  $\mu > \mu_F$ , the perturbative calculation is ok.
- if the scale is soft  $\mu < \mu_F$ , one should remove such an occurrence from the calculation and use an incoming quark  $H$  in the corrected/alternative calculation.



Summing this up one thus gets three contributions to the total cross-section:

• The perturbatively calculated process one started with:

$$
\sigma_{AB\to X\bar{H}} = f_{g/A} \otimes \hat{\sigma}_{gs\to X\bar{H}}^{(n+1)} \otimes f_{s/B}.
$$

• Process an order lower in  $\alpha_s$  but with an incoming quark  $H$ :

$$
\sigma_{AB\to X}=f_{H/A}\otimes\hat{\sigma}_{H_S\to X}^{(n)}\otimes f_{s/B}.
$$

• An appropriate subtraction contribution  $\sigma_{sub}$  the form of which needs to be determined.

## Short derivation of the subtraction terms:

The appropriate subtraction terms can actually be derived from the factorization theorem itself by using DGLAP at the parton level and doing power counting of  $\alpha_s$  (there are other ways like formal  $\overline{\rm MS}$  in D dimensions):

• The pQCD cross-section  $\sigma_{\rm ab\to X}$  involving initial state partons  ${\rm a,b}$  is subject to the same factorization theorem:  $\sigma_{ab\to X}=\sum f_{c/a}\otimes\hat{\sigma}_{cd\to X}\otimes f_{d/b},$ 

$$
\sigma_{ab\to X}=\sum_{c,d}f_{c/a}\otimes\hat{\sigma}_{cd\to X}\otimes f_{d/b},
$$

• At zero-th order in  $\alpha_s$  (0 = lowest possible order):

$$
f_{i/j}^{(0)}(\xi) = \delta_j^i \delta(\xi - 1)
$$

• and hence:

$$
\sigma_{ab \to X}^{(0)} = \hat{\sigma}_{ab \to X}^{(0)}.
$$

Subsequently, at first order in  $\alpha_s$  recursively from DGLAP:

$$
f_{i/j}(\xi) = f_{i/j}^{(0)}(\xi) + f_{i/j}^{(1)}(\xi) = f_{i/j}^{(0)}(\xi) + \frac{\alpha_s(\mu_F)}{2\pi} P_{j \to i}^{(0)}(\xi) \ln\left(\frac{\mu_F^2}{m^2}\right),
$$

• and thus at this order:

$$
\sigma_{ab\to X}^{(1)} = \hat{\sigma}_{ab\to X}^{(1)} + \sum_c f_{c/a}^{(1)} \otimes \hat{\sigma}_{cb\to X}^{(0)} + \sum_d \hat{\sigma}_{ad\to X}^{(0)} \otimes f_{d/b}^{(1)},
$$

• The last equation can thus be inverted to give: $\hat{\sigma}^{(1)}_{ab \to X} = \sigma^{(1)}_{ab \to X} - \sum_{i,j}$ 

be inverted to give:  
\n
$$
\hat{\sigma}_{ab \to X}^{(1)} = \sigma_{ab \to X}^{(1)} - \sum_{c} f_{c/a}^{(1)} \otimes \hat{\sigma}_{cb \to X}^{(0)} - \sum_{d} \hat{\sigma}_{ad \to X}^{(0)} \otimes f_{d/b}^{(1)},
$$

• Putting it back into the factorization theorem expression:

$$
\sigma_{AB \to X} = \sigma_{AB \to X}^{(0)} + \sigma_{AB \to X}^{(1)} - \sigma_{AB \to X}^{\text{subt}},
$$

• with the subtraction terms given by:



Other approaches can give slightly different subtraction terms however this one has certain advantages as will be shown.

## Meeting the Experimental Requirements (not really <sup>a</sup> digression)

- While experimentalists might be thankful for accurate total cross-section predictions what they really need are simulated events which amounts to fully differential cross-sections.
- Furthermore, they are not interested in inclusive final states  $(X)$  but rather *exclusive* occurrences (e.g. observing a quark H at a certain  $p_T$ /energy...)

## How can we simulate an (part) exclusive final state, e.g.  $X + H$ ?

- Generating events from pQCD model involving H in the final state, produced e.g. in gluon splits like in our test case.
- Or Pick a specific (back) evolution to gluon from a process with incoming H, the probability to obtain an additional H at a certain energy/scale/ $p_T$  $= \mu$  is given through the Sudakov exponent (commonly known as parton showering):

$$
S_a = \exp\left\{-\int_{\mu^2}^{\mu_P^2} \frac{d\mu'^2}{\mu'^2} \frac{\alpha_s(\mu'^2)}{2\pi} \times \sum_c \int_{\xi_c}^1 \frac{dz}{z} P_{a \to c}(z) \frac{f_{a/I}(\frac{\xi_c}{z}, \mu'^2)}{f_{c/I}(\xi_c, \mu'^2)}\right\}.
$$



• The latter procedure accurate in the collinear region (no divergencies) by construction (resummation). By definition the evolution variable  $\mu$ is limited by  $\mu_F$  from above.

## Parton Showering vs. Matrix Elements (pQCD)

There are two down-sides to this procedure:

- The first approac<sup>h</sup> needs <sup>a</sup> subtraction term (on an event-by-event basis).
- The second approach leads to the same final state thus it double-counts the first one at least in a certain region of phase space.
- However these two deficiencies can be made to cancel each other out.
	- There are several ways of doing this, generally one needs to tune the shower evolution variable  $\mu$  to match the scales/virtualities in pQCD calculation (also previously marked  $\mu$  on purpose) and applying suitable kinematic mappings (generally <sup>a</sup> hard task).
	- In our approach we adapted and generalized the prescription suggested by Collins et al [hep-ph/0110257,hep<sup>p</sup>h/0001040,hep-ph/0105291] where they have derived <sup>a</sup> consistent procedure of combining <sup>a</sup> few select processes (e.g. the Drell-Yan  $q\bar{q}\to Z$  with  $gq\to Z$   $q$ ), always/only for gluons splits in the initial state at Born/tree level: The flip side is that 'consistent' means the method reproduces the Compton part of the NLO differential cross-section exactly, by paper calculation.

#### Parton Showering vs. Matrix Elements (pQCD) cont'd

Let us see how this works:

• The fully differential parton-showered differential cross-section is given by:

$$
d\sigma_{AB\to X\bar{H}} = dS_{g\to H}(\mu) f_{H/A}(\mu_F) d\hat{\sigma}_{Hs\to X}^{(n)} f_{s/B}(\mu_F),
$$

• And our derived subtraction term is:

$$
d\sigma_{\text{subt}} = f_{g/A}(\mu_F) df_{H/g}^{(1)}(\mu) d\hat{\sigma}_{\text{Hs}\to X}^{(n)} f_{s/B}(\mu_F).
$$

• Taking the limit  $\mu \rightarrow \mu_F$  one quickly sees that:

$$
dS_{g\to H}(\mu) f_{H/A}(\mu_F) \xrightarrow{\mu \to \mu_F} f_{g/A}(\mu_F) \xrightarrow{ \alpha_s(\mu_F)} P_{g\to H} d\Phi
$$
  

$$
f_{g/A}(\mu_F) df_{H/g}^{(1)}(\mu) \xrightarrow{\mu \to \mu_F} f_{g/A}(\mu_F) \xrightarrow{\alpha_s(\mu_F)} P_{g\to H} d\Phi,
$$

with  $d\Phi$  denoting all the variables in the differential.

- In this limit  $\mu \to \mu_F$  the two terms thus cancel *on paper*, i.e. exactly.
- In the other (collinear  $\mu \to m_H$ ) limit the subtraction term cancels its parent expression by construction.
- The subtraction term is thus supposed to interpolate between the two contributions while removing the overlap (double counting), resulting in a *smooth* combination of the two approaches.

## Parton Showering vs. Matrix Elements (pQCD) cont'd

- This procedure requires <sup>a</sup> specific form of parton showering and kinematic transforms which leads to the corollary that subsequently the PDFs for quarks need to be modified [hep-ph/0204127] (the subtraction procedure is formally not equal to standard subtraction schemes like  $\overline{\text{MS}}$ ).
- In order to match the derived prescription with the explicit  $\overline{\rm MS}$  NLO result for Z + jet production on paper new PDFs need to be defined:

$$
z f_{i/I}^{JCC}(z, \mu^2) = z f_{i/I}^{\overline{\text{MS}}}(z, \mu^2)
$$
  
+ 
$$
\frac{\alpha_s(\mu^2)}{2\pi} \int_{z}^{1} d\xi \frac{z}{\xi} f_{g/I}^{\overline{\text{MS}}}(\xi, \mu^2) \left[ P_{g \to i\bar{i}}(\frac{z}{\xi}) \ln\left(1 - \frac{z}{\xi}\right) + \frac{z}{\xi} \left(1 - \frac{z}{\xi}\right) \right]
$$
  
+ 
$$
\mathcal{O}(\text{first-order quark terms}) + \mathcal{O}(\alpha_s^2)
$$

- This simple form is *particular* to the proscribed kinematic mapping/showering, it is *not general*!
- Side comment: This means that the discussion of NLO vs LO PDFs for showering is actually more complicated.
- These new distributions can in <sup>a</sup> reasonably straightforward manner be obtained by numerical integration using e.g. CTEQ functions as input.

## Massive Partons in the Initial states

- The second point is the treatment of quark masses of initial state partons:
	- All the partons in 'usual' NLO calculations are generally treated as massless.
	- This becomes a conceptual problem in case of gluon splitting to heavy partons like  $b$  or  $c$  quarks.
	- The heavy partons in the final state need to have masses to accurately describe the observable jet kinematics.
	- the ACOT (M. A. G. Aivazis, J. C. Collins, F. I. Olness and W. K. Tung) [hep-ph/9312318, hep-ph/9312319] and many derived papers provide <sup>a</sup> method of incorporating the massive quarks into the Factorization Theorem (actually done for DIS and <sup>c</sup> quark). There is <sup>a</sup> lot of work done on this, for the impact on LHC have <sup>a</sup> look at [hep-ph/9712494] and papers citing it.
- We tried to merge this method with the Collins' method and applied it to <sup>a</sup> few LHC-related cases.
	- The method takes us back to basics of the treatment of masses in the factorization theorem:
	- The factorization theorem is actually derived using the light-cone coordinates  $p^\mu = (p^+, \vec{p}^T, p^-)$  where  $p^\pm =$  $\frac{1}{\sqrt{2}}(\mathrm{p}^0\pm \mathrm{p}^3)$ , which can incorporate particle masses.
	- This translates to modified kinematics in factorization w.r.t. massless:  $p_a=(p^+_a,\vec{0}^T,p^-_a)=(\xi_aP^+_A,\vec{0}^T,\frac{m^2_a}{2\xi_aP^+_A})$  and  $p_b = (p_b^+, \vec{0}^T, p_b^-) = (\frac{m_b^2}{2\xi_b P_B^-}, \vec{0}^T, \xi_b P_B^-)$  for the colliding a and b partons.
	- We adapted the method into a Monte-Carlo algorithm for the proton-proton collisions.

## Massive Partons in the Initial states cont'd

• Very recently we now added massive corrections to the splitting kernel  $P_{g\rightarrow H}$ :

$$
P_{g \to H}^{\text{massive}} = P_{g \to H}^{\text{massless}} + P_{g \to H}^{\text{correction}} = \mathcal{T}_{\text{R}} \left( 1 - 2z(1 - z) \right) + \mathcal{T}_{\text{R}} \left( \frac{2z(1 - z)m_H^2}{p_T^2 + m_H^2} \right),
$$

which further improves our method in the very collinear region  $(\mu \to m_H)$ .

• We are the first ones to do it by adapting the calculations from Catani et al paper [hep-ph/0201036] and the results go in the right direction.

## Implementation: AcerMC 3.x Monte-Carlo generator

- A Monte-Carlo generator of select Standard Model processes for searches at ATLAS
- Matrix element coded by MADGRAPH/HELAS
	- ➔T. Stelzer and W. F. Long, Comput.Phys.Commun. <sup>81</sup> (1994) 357.
- Phase space sampling done by native **AcerMC** routines: ➔Eur. Phys. J. C 439-450 (2005)

<sup>⊕</sup> Each channel topology constructed from generic t-type and s-type modules and massive sampling functions. The event topologies auto-generated from modified MADGRAPH/HELAS code.

## <sup>⊕</sup> multi-channel approach

- ➔J.Hilgart, R. Kleiss, F. Le Dibider, Comp. Phys. Comm. <sup>75</sup> (1993) 191.
- ➔F. A. Berends, C. G. Papadopoulos and R. Pittau, hep-ph/0011031.
- <sup>⊕</sup> additional ac-VEGAS smoothing

➔G.P. Lepage, J. Comput. Phys. <sup>27</sup> (1978) 192.

• ac-VEGAS Cell splitting in view of maximal weight reduction based on function:

$$
\langle F \rangle_{\text{cell}} = \left(\Delta_{\text{cell}} \cdot \text{wt}_{\text{cell}}^{\text{max}}\right) \cdot \left\{1 - \frac{\langle \text{wt}_{\text{cell}} \rangle}{\text{wt}_{\text{cell}}^{\text{max}}}\right\}
$$

- ac-VEGAS logic in this respect analogous to FOAM:
- ➔S. Jadach, Comput. Phys. Commun. <sup>130</sup> (2000) 244.





## Various types:

- Processes involving top pair production
- The single top processes
- The Z-prime decay to tops
- $\bullet$  The  $(A+B)$  denote PS+ME matched processes.
- All processes with decayed tops include full spin information.

## Example of 2  $\rightarrow$  4 processes: ud  $\rightarrow$  W<sup>+</sup>g<sup>\*</sup>  $\rightarrow$  l<sup>+</sup> $\nu_1$ bb, pp @ 14 TeV

• Examples of invariant mass distributions obtained with  $\text{AcerMC}$ 



• Some variances and unweighing efficiencies obtained using standard  $\text{AcerMC 1.4}$  and new  $\text{AcerMC 2.0}$ (and later) <sup>p</sup>hase space sampling.



## Example of 2 → 6 processes:  $gg \rightarrow b\bar{b}W^+W^- \rightarrow b\bar{b}\ell\bar{\nu}_\ell\bar{\ell}\nu_\ell$

• The process cross-sections and variances with their uncertainties and unweighing efficiencies as obtained for two sample  $2 \rightarrow 6$  processes implemented in  ${\bf AcerMC}$  2.0 Monte–Carlo generator.



• Example of the weight distributions obtained with the two processes.



• Bottom line is: It Works!

## Massive Parton Shower + pQCD Implementation:

• The AcerMC MC generator now incorporates the described procedure of Parton Shower (ISR) and ME matching for  $g \to b\bar{b}$  splitting for a small set of processes

## Details:

- The ISR 'showering' involving  $g \to b\bar{b}$  has been implemented inside AcerMC.
- This algorithm is used to evolve a process from  $bX \to Y$  to  $gX \to Y \oplus b.$
- This process is combined with the corresponding 'NLO' process  $gX \to Y + b$  and the double counting terms are calculated and subtracted on event-by-event basis.
- As the result a fraction of events has negative  $(=1)$  weights!
- This procedure has been implemented for the:
	- $-$  t-channel single top production.
	- $-\ b\bar b WW$  production which involves the ('evolved')  $tWb$  single top production.
	- Associated  $Z^0 b$  production.

## t-channel single top production:

- The t-channel process is the combined production of the  $qb \rightarrow qt$  and  $qg \rightarrow qtb$  W-exchange processes.
- One needs to remove the double counting between the ISR  $g\,\to\,b\bar{b}$  splitting and the next-order  $\alpha_S$  process  $qg \rightarrow qtb$ .
- In fact the t-channel single top production involves the full matrix element including top decays.



**tW-channel single top production:** Similar case, it double counts the  $tWb$  diagrams.



## Kinematic distributions for t-channel single top :

- Note that <sup>a</sup> smooth continuation in the b-quark virtuality is achieved.
- The  $p_T$  distribution is again a result of non-trivial contributions.



## Kinematic distributions for tW-channel single top:

- Note that <sup>a</sup> smooth continuation in the b-quark virtuality is again achieved.
- The  $p_T$  distribution again a result of non-trivial contributions.
- The plots serve as a cross-check; in AcerMC process 20 the procedure is applied to the  $WWb\bar{b}$   $(2\to 6)$  process 13 which includes the  $tWb$  intermediate states among its 31 diagrams.



## Drell-Yan  $Z + b$  production:

- The double counting is in this case two-fold: either b or  $\bar{b}$  can originate in gluon splitting.
- In fact the Drell-Yan case has been implemented with the full matrix element including <sup>p</sup>hoton interference.





#### Drell-Yan  $Z + b$  production cont'd:

- Note that <sup>a</sup> smooth continuation in the b-quark virtuality is achieved regardless of the matching point/factorization scale.
- The  $p_T$  distribution is a result of non-trivial contributions in this case.



## Drell-Yan  $Z + b$  production cont'd: massive evolution kernels (new)

- In the new (upcoming) **AcerMC** version the new massive evolution kernel is introduced.
- Note that the kink at low  $\mu$  scale now disappears, the continuation really smooth...



## Even Newer Drell-Yan  $gg \to Zb\bar{b}$

- In the new (upcoming) AcerMC version the a new  $\alpha_s^2$  process of associated  $\rm gg\to Zb\bar{b}$  production with overlap removal.
- A proof-of-concept that the method is indeed *iterative* in nature





# Even Newer Drell-Yan  $gg \to Zb\bar{b}$  cont'd

• The distributions again smooth in one dimension. . .



# Even Newer Drell-Yan  $gg \to Zb\bar{b}$  cont'd

- ... and in two dimensions!
- Plots show separate pQCD order contributions with counter-terms.



• Very preliminary!

# Even Newer Drell-Yan  $gg \to Zb\bar{b}$  cont'd

• Plots show incremental sums of pQCD order contributions with counter-terms.



 $\alpha_{\rm s}^0$  + 2 Parton showers  $\alpha_{\rm s}^0 \oplus \alpha_{\rm s}^1$   $\alpha_{\rm s}^0 \oplus \alpha_{\rm s}^1 \oplus \alpha_{\rm s}^2$ 

• Very preliminary!

## Conclusions:

- The described procedure has been shown to work. . .
- For details <sup>p</sup>lease consult [hep-ph/0603068] or JHEP 0609:033,2006.
- In case one wants to check this in practice: The complete **AcerMC** manual available from:

http://cern.ch/Borut.Kersevan/AcerMC.Welcome.html

- **AcerMC** code is available from the same URL.
	- This procedure is recursive, so it could be implemented for arbitrary number of splits (ISR/FSR) and possibly a CKKW-like procedure [hep-ph/0109231] could be achieved.
	- the  $Z^0 b\bar b$  paper in preparation.
	- Needs work and time...