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The breakdown of collinear factorization in QCD

Jeff Forshaw

In collaboration with Mike Seymour, Albrecht Kyrieleis & Andrzej Siodmok

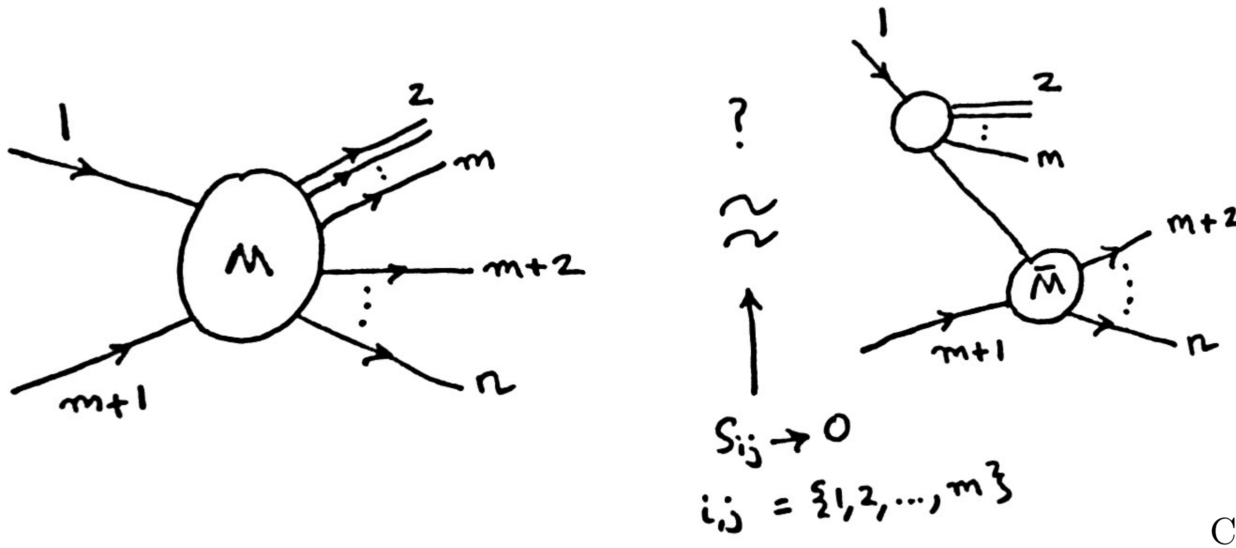
Question
Is it possible to factorize all collinear singularities into process independent (universal) functions?

Answer
Only for sufficiently inclusive observables or for processes involving zero (e^+e^-) or one (DIS) incoming quark or gluon.

Collins, Soper, Sterman
Nucl. Phys B308 (1988) 833.

This talk
How does the anticipated breakdown arise in perturbation theory?

Note: collinear factorization is assumed in all of the Monte Carlo event generators. Factorization breaking effects are included in a very phenomenological fashion (i.e. underlying event via multi-parton interactions or colour reconnections).



Catani, de Florian, Rodrigo
arXiv:1112.4405

$$|M\rangle \approx \boldsymbol{Sp}(p_1, ..., p_m; \tilde{P}; p_{m+1} ..., p_n)|\overline{M}\rangle$$

All orders.
Corrections are $\sqrt{s_{ij}}$ suppressed.

Even at one-loop it is not the case that

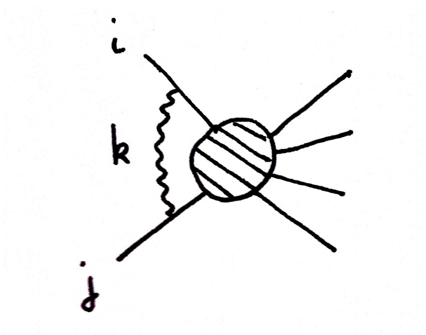
$$|M\rangle \approx \boldsymbol{Sp}(p_1, ..., p_m; \tilde{P})|\overline{M}\rangle$$

But the factorization breaking terms can cancel in $\langle M|M\rangle$.

This "generalized factorization" is consistent with the known factorization of all infra-red poles in QCD amplitudes.

Accurate up to finite non-logarithmic corrections, i.e. contains all IR poles and their associated logarithms.

Divergent structure of QCD: Loops



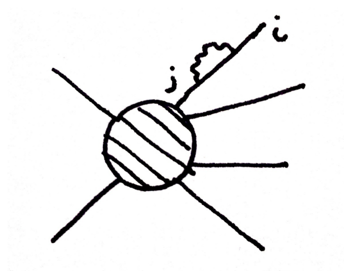
soft

$$\mathrm{Re}\ \boldsymbol{I}^{(1)}_{\mathrm{V,soft}} = \sum_{i\neq j} \frac{\alpha_s}{4\pi} \boldsymbol{T}_i \cdot \boldsymbol{T}_j \int_0^{s_{ij}} \frac{dk_\perp^2}{(k_\perp^2)^{1+\epsilon}} \mu^{2\epsilon} \int_{\frac{k_\perp^2}{s_{ij}}}^1 \frac{dz}{z}$$

$$\mathrm{Im}\ \boldsymbol{I}^{(1)}_{\mathrm{V,soft}} = \sum_{i\neq j} \frac{\alpha_s}{4\pi} \boldsymbol{T}_s^2 \int_0^{s_{ij}} \frac{dk_\perp^2}{(k_\perp^2)^{1+\epsilon}} \mu^{2\epsilon}\ 2\pi$$

Coulomb/Glauber

$$\boldsymbol{T}_s = \boldsymbol{T}_1 + \boldsymbol{T}_{m+1} = -\sum_{i\neq 1,m+1}^{n} \boldsymbol{T}_i$$



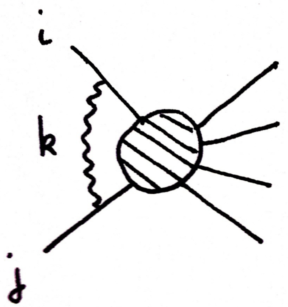
hard collinear

$$\boldsymbol{I}^{(1)}_{\mathrm{V,hard}} = \sum_{i} \frac{\alpha_s}{4\pi} \int_0^{\mu_i^2} \frac{dk_\perp^2}{(k_\perp^2)^{1+\epsilon}} \mu^{2\epsilon} \sum_j \int_0^1 dz\, P_{ij}(z)$$

$$\begin{aligned} \text{e.g.}\quad P_{qq} &= -\boldsymbol{T}_q^2\,(1+z) \\ P_{qq}^{\mathrm{full}} &= \boldsymbol{T}_q^2 \left(\frac{1+z^2}{(1-z)_+} + \frac{3}{2}\delta(1-z) \right) \\ &= \boldsymbol{T}_q^2 \left(\frac{2}{(1-z)_+} + \frac{3}{2}\delta(1-z) + P_{qq} \right) \end{aligned}$$

$$P_{gq} = -\boldsymbol{T}_q^2 \left(\frac{1+(1-z)^2}{z} - \frac{2}{z} \right)$$

$$|M^{(1)}\rangle = \boldsymbol{I}^{(1)} |M^{(0)}\rangle + |M^{(1)\mathrm{fin.}}\rangle$$

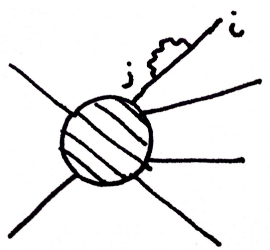


soft

$$\mathrm{Re}\,\boldsymbol{I}^{(1)}_{\mathrm{V,soft}} \sim \sum_{i\neq j} \boldsymbol{T}_i\cdot\boldsymbol{T}_j \int_0^{s_{ij}} \frac{dk_\perp^2}{(k_\perp^2)^{1+\epsilon}}\mu^{2\epsilon}\int_{\frac{k_\perp^2}{s_{ij}}}^1 \frac{dz}{z}$$

$$= \sum_{i\neq j} \boldsymbol{T}_i\cdot\boldsymbol{T}_j \left(\frac{\mu^2}{s_{ij}}\right)^{\epsilon}\frac{1}{\epsilon^2}$$

$$= -\sum_i \boldsymbol{T}_i^2\,\frac{1}{\epsilon^2} - \sum_{i\neq j}\boldsymbol{T}_i\cdot\boldsymbol{T}_j\,\frac{1}{\epsilon}\ln\frac{s_{ij}}{\mu^2} + \text{finite}$$



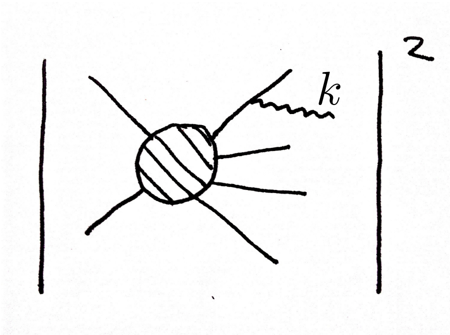
hard collinear

$$\boldsymbol{I}^{(1)}_{\mathrm{V,hard}} \sim -\sum_i \left(\frac{\mu^2}{\mu_i^2}\right)^{-\epsilon}\frac{1}{\epsilon}\gamma_i$$

$$= -\frac{\gamma_i}{\epsilon} + \text{finite}$$

$$d\sigma_V^{(1)} = \langle M^{(0)}|I_V^{(1)} + I_V^{(1)\dagger}|M^{(0)}\rangle\, d(PS)_n$$

Divergent structure of QCD: Real emission



$$d\sigma_R^{(1)} = -\langle M^{(0)}|\boldsymbol{Sp}^{(0)\dagger}\boldsymbol{Sp}^{(0)}|M^{(0)}\rangle\,\Phi(\{p_i\})\,d(PS)_{n+1}$$

$$\int \boldsymbol{Sp}^{(0)\dagger}\boldsymbol{Sp}^{(0)}\,d(PS)_k = I_V^{(1)} + I_V^{(1)\dagger}$$

$$\Phi(\{p_i\}) = 1 \rightarrow \text{perfect cancellation} = \text{KLN/Bloch-Nordsieck}$$

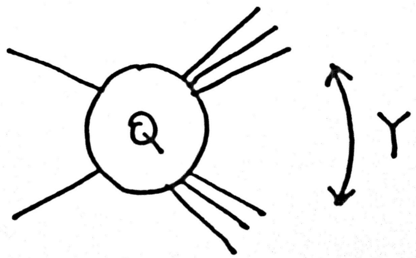
$$(I_V^{(1)} + I_V^{(1)\dagger}) \otimes \Phi(\{p_i\}) = \frac{\alpha_s}{4\pi}\mu^{2\epsilon}\int \frac{dk_\perp^2}{(k_\perp^2)^{1+\epsilon}}\frac{dz}{z}(\boldsymbol{T}_i\cdot\boldsymbol{T}_j + zP(z))\,\Phi(\{p_i\}))$$

For a general observable only the poles need to cancel (IRC safety).

The remnant of this cancellation is uncancelled logarithms.

Note: the imaginary part of the loops needs a different mechanism to cancel.

An example: dijet production with a veto



Veto in-gap real emission with $k_\perp > Q_0$

Emitted "out of the gap"

$$(I_V^{(1)} + I_V^{(1)\dagger}) \otimes \Phi(\{p_i\}) \quad \sim \quad 2\mu^{2\epsilon} \int_0^{Q^2} \frac{dk_\perp^2}{(k_\perp^2)^{1+\epsilon}} \int_{k_\perp^2/Q^2}^{e^{-Y/2}} \frac{dz}{z} (\boldsymbol{T}_i \cdot \boldsymbol{T}_j)$$

$$+ \quad 2\mu^{2\epsilon} \int_0^{Q_0^2} \frac{dk_\perp^2}{(k_\perp^2)^{1+\epsilon}} \int_{e^{-Y/2}}^{1} \frac{dz}{z} (\boldsymbol{T}_i \cdot \boldsymbol{T}_j) + \cdots$$

Emitted "in the gap"

Putting $s_{ij} \sim Q^2 > Q_0^2$ for i and j on opposite sides of the gap.

Adding to the loop correction gives

$$d\sigma_V^{(1)} + d\sigma_R^{(1)} = \langle M^{(0)} | \boldsymbol{T}_L \cdot \boldsymbol{T}_R | M^{(0)} \rangle \, d(PS)_n \, \frac{\alpha_s}{2\pi} Y \log \frac{Q^2}{Q_0^2}$$

This is simply the loop correction integrated over the region of "phase-space" where the real emission is vetoed.

Beyond one-loop:

$$\Big| M^{(l)} \Big\rangle = \sum_{l'=1}^{l} \boldsymbol{I}^{(l')} \Big| M^{(l-l')} \Big\rangle + \Big| M^{(l)}_{\text{finite}} \Big\rangle ,$$

$$\Big| M \Big\rangle = \boldsymbol{I} \Big| M \Big\rangle + \Big| M_{\text{finite}} \Big\rangle ,$$

A remarkable result:

The IR operator to all orders has (almost?) the same form as the exponentiated one-loop result.

$$\boldsymbol{Z} = \exp\left(-\tilde{\boldsymbol{I}} \right)$$

$$|M_{\text{finite}}\rangle = \boldsymbol{Z}\, |M\rangle$$

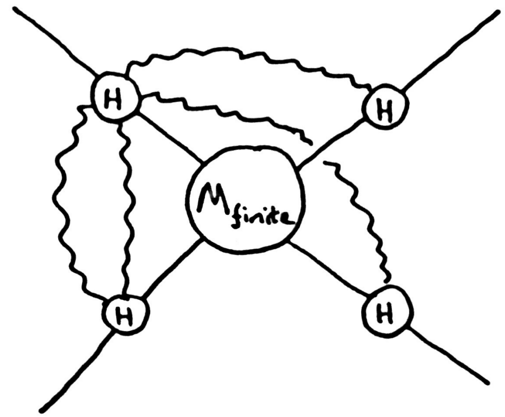
$$1 - \boldsymbol{I} \equiv \exp(-\tilde{\boldsymbol{I}})$$

Dixon, Magnea, Sterman
arXiv:0805.3515;
Gardi & Magnea
arXiv:0901.1091
Becher & Neubert
arXiv:0903.1126

This is incidental for us.

We are more interested in the factorization of collinear poles...

$$|M_{\text{finite}}\rangle = \boldsymbol{Z}\,|M\rangle$$



At this level there is no collinear factorization because soft gluons link all external partons.

One loop again

$$\langle M^{(0)}|M^{(1)}\rangle + \text{h.c.} \;=\; \langle \overline{M}^{(0)}|\boldsymbol{Sp}^{(0)\dagger}\,\boldsymbol{Sp}^{(1)}|\overline{M}^{(0)}\rangle + \text{h.c.}$$
$$+\; \langle \overline{M}^{(0)}|\boldsymbol{Sp}^{(0)\dagger}\,\boldsymbol{Sp}^{(0)}|\overline{M}^{(1)}\rangle + \text{h.c.}$$

The singularities of the one-loop splitting matrix, defined by

$$\boldsymbol{Sp}^{(1)} = \boldsymbol{I}_C^{(1)}\,\boldsymbol{Sp}^{(0)} + \boldsymbol{Sp}^{(1)\text{fin.}} ,$$

can be extracted using

$$\boldsymbol{I}_C^{(1)}\,\boldsymbol{Sp}^{(0)} = \boldsymbol{I}^{(1)}\,\boldsymbol{Sp}^{(0)} - \boldsymbol{Sp}^{(0)}\,\overline{\boldsymbol{I}}^{(1)} ,$$

which we write as

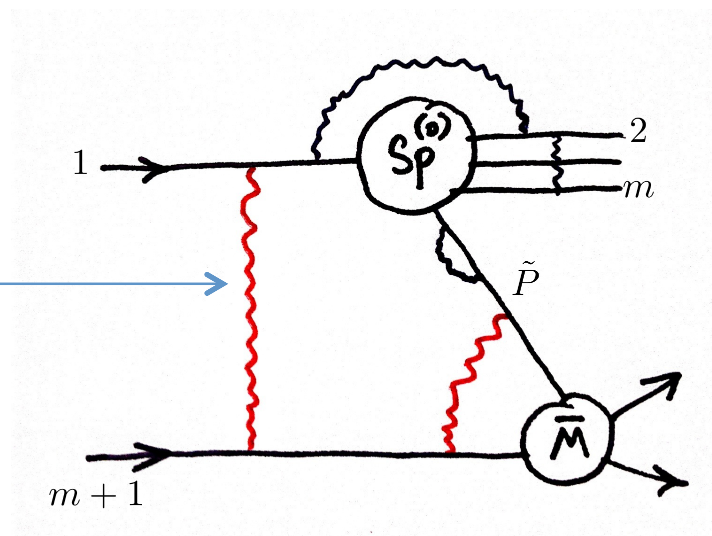
$$\boldsymbol{I}_C^{(1)} = \boldsymbol{I}^{(1)} - \overline{\boldsymbol{I}}^{(1)}.$$

After some colour algebra:

$$\boldsymbol{I}_C^{(1)} = \frac{\alpha_s}{2\pi}\,\frac{1}{2}\Bigg\{\left(\frac{1}{\epsilon^2}C_{\widetilde{P}} + \frac{1}{\epsilon}\gamma_{\widetilde{P}}\right) - \sum_{i=1}^{m}\left(\frac{1}{\epsilon^2}C_i + \frac{1}{\epsilon}\gamma_i - \frac{2}{\epsilon}C_i \ln|z_i|\right) - \frac{i\pi}{\epsilon}\left(C_{\widetilde{P}} - C_1 + \sum_{i=2}^{m} C_i\right)$$
$$- \frac{1}{\epsilon}\sum_{\substack{i,\ell=1\\ i\neq\ell}}^{m} \boldsymbol{T}_i\cdot\boldsymbol{T}_\ell \ln\frac{|s_{i\ell}|}{|z_i|\,|z_\ell|\,\mu^2}\Bigg\} + \widetilde{\boldsymbol{\Delta}}_C^{(1)},$$

with

$$\widetilde{\boldsymbol{\Delta}}_C^{(1)} = \frac{\alpha_s}{2\pi}\left\{2\times\frac{i\pi}{\epsilon}\boldsymbol{T}_{m+1}\cdot\left(\boldsymbol{T}_1 - \boldsymbol{T}_{\widetilde{P}}\right)\right\}.$$



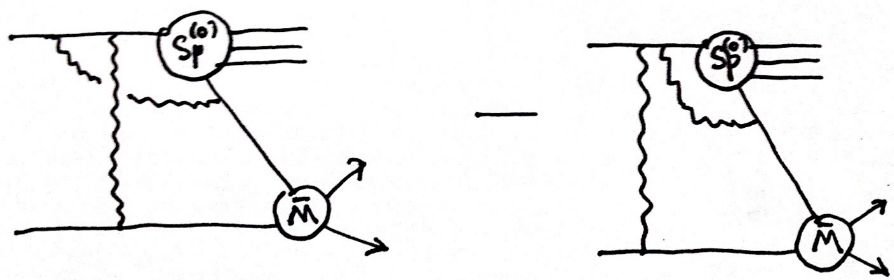
Coulomb exchange in the initial state breaks amplitude level factorization at one-loop level. It trivially **cancels** at the cross-section level.

Since $\widetilde{\boldsymbol{\Delta}}_C^{(1)}$ is anti-Hermitian

Two loops

$$\langle \overline{M}^{(0)} | \mathbf{P}^{(2)}_{\text{n.f.}} | \overline{M}^{(0)} \rangle$$

$$\mathbf{P}^{(2)}_{\text{n.f.}} = \frac{1}{2} \boldsymbol{Sp}^{(0)\dagger} \left[\left(\overline{\boldsymbol{I}}^{(1)} + \overline{\boldsymbol{I}}^{(1)\dagger} + \boldsymbol{I}_C^{(1)\text{fact.}} + \boldsymbol{I}_C^{(1)\text{fact.}\dagger} \right), \widetilde{\boldsymbol{\Delta}}_C^{(1)} \right] \boldsymbol{Sp}^{(0)}$$



The factorization breaking still **cancels** at the cross-section level in pure QCD processes.

$$\mathrm{Tr}\Big[\left(|M^{(0)}\rangle \langle M^{(0)}| \right) \; [\boldsymbol{I}, \widetilde{\boldsymbol{\Delta}}_C^{(1)}] \Big] = 0$$

$[\boldsymbol{I}, \widetilde{\boldsymbol{\Delta}}_C^{(1)}]$ is hermitian and a colour basis exists in which it is **anti-symmetric**.

$\mathbf{H} = |M^{(0)}\rangle \langle M^{(0)}|$ is real and **symmetric** in the same basis.

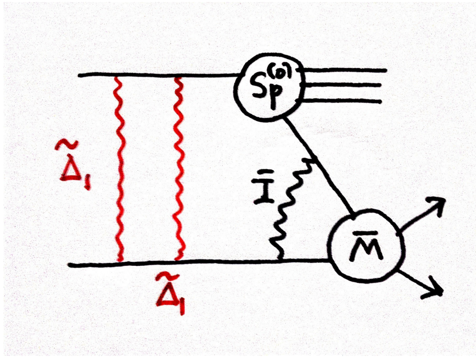
Seymour & Sjödahl

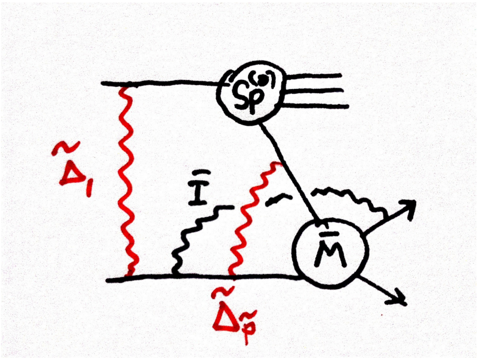
arXiv:0810.5756

Three loops

$$\langle \overline{M}^{(0)} | \mathbf{P}^{(3)}_{\text{n.f.}} | \overline{M}^{(0)} \rangle$$

$$\begin{aligned}\mathbf{P}^{(3)}_{\text{n.f.}} \sim \quad & \frac{1}{6} \boldsymbol{Sp}^{(0)\dagger} \left(\left[\widetilde{\boldsymbol{\Delta}}^{(1)}_{1}, \left[\widetilde{\boldsymbol{\Delta}}^{(1)}_{1}, \overline{\boldsymbol{I}}^{(1)} + \overline{\boldsymbol{I}}^{(1)\dagger} \right] \right] - \left[\widetilde{\boldsymbol{\Delta}}^{(1)}_{\widetilde{P}}, \left[\widetilde{\boldsymbol{\Delta}}^{(1)}_{\widetilde{P}}, \overline{\boldsymbol{I}}^{(1)} + \overline{\boldsymbol{I}}^{(1)\dagger} \right] \right] \right) \boldsymbol{Sp}^{(0)} \\ & + \frac{1}{2} \boldsymbol{Sp}^{(0)\dagger} \left(\left[\widetilde{\boldsymbol{\Delta}}^{(1)}_{\widetilde{P}}, \left[\widetilde{\boldsymbol{\Delta}}^{(1)}_{\widetilde{P}}, \overline{\boldsymbol{I}}^{(1)} + \overline{\boldsymbol{I}}^{(1)\dagger} \right] \right] - \left[\widetilde{\boldsymbol{\Delta}}^{(1)}_{\widetilde{P}}, \left[\widetilde{\boldsymbol{\Delta}}^{(1)}_{1}, \overline{\boldsymbol{I}}^{(1)} + \overline{\boldsymbol{I}}^{(1)\dagger} \right] \right] \right) \boldsymbol{Sp}^{(0)}.\end{aligned}$$





This gives a **non-zero** contribution to the cross-section in QCD.

It will cancel against real emission graphs for sufficiently inclusive observables, i.e. cuts through the "eikonal" gluon (in black).

JRF, Seymour, Siodmok
arXiv:1206.6363

These factorization breaking terms induce double-logarithmic corrections at fourth-order in the "**dijet plus a veto**" observable.

$$\frac{\sigma_{\mathrm{SLL}}(Q, Q_0)}{\sigma_0} \sim \alpha_s^4 \log^5\left(\frac{Q}{Q_0}\right)\pi^2 Y$$

Kyrieleis, Seymour, JRF
arXiv:0808.1269
arXiv:hep-ph/0604094

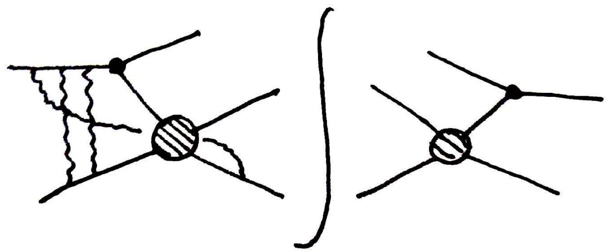
One collinear splitting

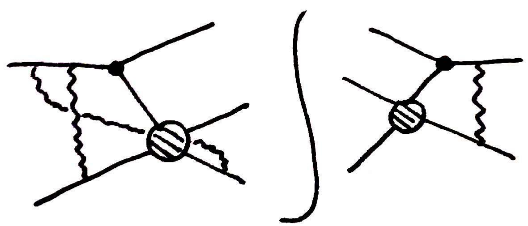
$$\sigma_1 = -\frac{2\alpha_s}{\pi}\int_{Q_0}^{Q}\frac{dk_T}{k_T}\left(2\ln\frac{Q}{k_T}\right)\left\langle m_0\right| e^{-\frac{2\alpha_s}{\pi}\int_{k_T}^{Q}\frac{dk'_T}{k'_T}\left(\frac{1}{2}Y\mathbf{t}_t^2-i\pi\mathbf{t}_1\cdot\mathbf{t}_2\right)}$$

$$\Big\{\mathbf{t}_1^2 e^{-\frac{2\alpha_s}{\pi}\int_{Q_0}^{k_T}\frac{dk'_T}{k'_T}\left(\frac{1}{2}Y\mathbf{t}_t^2-i\pi\mathbf{t}_1\cdot\mathbf{t}_2\right)} e^{-\frac{2\alpha_s}{\pi}\int_{Q_0}^{k_T}\frac{dk'_T}{k'_T}\left(\frac{1}{2}Y\mathbf{t}_t^2+i\pi\mathbf{t}_1\cdot\mathbf{t}_2\right)}$$

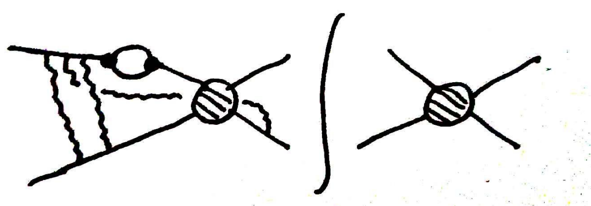
$$-\mathbf{t}_1^{a\dagger} e^{-\frac{2\alpha_s}{\pi}\int_{Q_0}^{k_T}\frac{dk'_T}{k'_T}\left(\frac{1}{2}Y\mathbf{T}_t^2-i\pi\mathbf{T}_1\cdot\mathbf{T}_2\right)} e^{-\frac{2\alpha_s}{\pi}\int_{Q_0}^{k_T}\frac{dk'_T}{k'_T}\left(\frac{1}{2}Y\mathbf{T}_t^2+i\pi\mathbf{T}_1\cdot\mathbf{T}_2\right)}\mathbf{t}_1^a\Big\}$$

$$e^{-\frac{2\alpha_s}{\pi}\int_{k_T}^{Q}\frac{dk'_T}{k'_T}\left(\frac{1}{2}Y\mathbf{t}_t^2+i\pi\mathbf{t}_1\cdot\mathbf{t}_2\right)}\left|m_0\right\rangle.$$





$$\{\ \}_2 = \left(\frac{i\pi Y}{2}\right)\left(-\frac{2\alpha_s}{\pi}\int_{Q_0}^{k_T}\frac{dk'_T}{k'_T}\right)^2\left\{\mathbf{t}_1^2\left[\mathbf{t}_t^2,\mathbf{t}_1\cdot\mathbf{t}_2\right]-\mathbf{t}_1^{a\dagger}\left[\mathbf{T}_t^2,\mathbf{T}_1\cdot\mathbf{T}_2\right]\mathbf{t}_1^a\right\}$$



$$\{\ \}_3 \equiv -\frac{Y\pi^2}{6}\left(-\frac{2\alpha_s}{\pi}\int_{Q_0}^{k_T}\frac{dk'_T}{k'_T}\right)^3\Big\{\mathbf{t}_1^2\Big[\left[\mathbf{t}_t^2,\mathbf{t}_1\cdot\mathbf{t}_2\right],\mathbf{t}_1\cdot\mathbf{t}_2\Big]$$

$$-\mathbf{t}_1^{a\dagger}\Big[\left[\mathbf{T}_t^2,\mathbf{T}_1\cdot\mathbf{T}_2\right],\mathbf{T}_1\cdot\mathbf{T}_2\Big]\mathbf{t}_1^a\Big\}.$$

One final step is missing: we here assume that the real + virtual cancellation fixes the limits on the integrals as above. It would be surprising if it were incorrect but it is not proven. It might change the numerical coefficient of the SLL.

Need to go beyond soft gluon approximation in collinear limit:

$$\int d^2k_T \int_{\text{out}} dy \quad \frac{d\sigma}{dyd^2k_T}\bigg|_{\text{soft}} \to \int d^2k_T \left[\int^{y_{\max}} dy \quad \frac{d\sigma}{dyd^2k_T}\bigg|_{\text{soft}} + \int_{y_{\max}}^{\infty} dy \quad \frac{d\sigma}{dyd^2k_T}\bigg|_{\text{collinear}} \right]$$

$$\int_{y_{\max}}^{\infty} dy \quad \frac{d\sigma}{dyd^2k_T}\bigg|_{\text{collinear}} = \int_{y_{\max}}^{\infty} dy \quad \left(\frac{d\sigma_{\text{R}}}{dyd^2k_T}\bigg|_{\text{collinear}} + \frac{d\sigma_{\text{V}}}{dyd^2k_T}\bigg|_{\text{collinear}} \right)$$

Soft approximation:

$$\int dz\,\frac{1}{2}\left(\frac{1+z^2}{1-z}\right) \to \int dy$$

Real collinear emission:

$$\int_{y_{\max}}^{\infty} dy \; \frac{d\sigma_{\mathrm{R}}}{dy d^2 k_T}\bigg|_{\text{collinear}} = \int_0^{1-\delta} dz \frac{1}{2}\left(\frac{1+z^2}{1-z}\right)\frac{q(x/z,\mu^2)}{q(x,\mu^2)} A_{\mathrm{R}}$$

$$= \int_0^{1-\delta} dz \frac{1}{2}\left(\frac{1+z^2}{1-z}\right)\left(\frac{q(x/z,\mu^2)}{q(x,\mu^2)} - 1\right) A_{\mathrm{R}} + \int_0^{1-\delta} dz \frac{1}{2}\frac{1+z^2}{1-z} A_{\mathrm{R}}$$

Virtual collinear emission:

$$\int_{y_{\max}}^{\infty} dy \; \frac{d\sigma_{\mathrm{V}}}{dy d^2 k_T}\bigg|_{\text{collinear}} = \int_0^{1-\delta} dz \frac{1}{2}\left(\frac{1+z^2}{1-z}\right) A_{\mathrm{V}}$$

$$y > y_{\max}$$

implies

$$\delta \approx \frac{k_T}{Q} \exp\left(y_{\max} - \frac{\Delta y}{2}\right)$$

If $A_R + A_V = 0$ then the divergence would cancel leaving behind a regularized splitting, which would correspond to the DGLAP evolution of the incoming quark pdf.

As we have seen, the Coulomb gluons spoil this cancellation. Instead we have

$$\int_0^{1-\delta} dz \frac{1}{2}\left(\frac{1+z^2}{1-z}\right)(A_{\mathrm{R}}+A_{\mathrm{V}}) = \ln\left(\frac{1}{\delta}\right)(A_{\mathrm{R}}+A_{\mathrm{V}}) + \text{subleading}$$
$$\approx \left(-y_{\max} + \frac{\Delta y}{2} + \ln\left(\frac{Q}{k_T}\right)\right)(A_{\mathrm{R}}+A_{\mathrm{V}})$$

Hence

$$\int_{Q_0}^{Q} \frac{dk_T}{k_T} \int_{\mathrm{out}} dy \to \int_{Q_0}^{Q} \frac{dk_T}{k_T} \left(\int^{y_{\max}} dy + (-y_{\max} + \ln \frac{Q}{k_T} \right) = \boxed{\frac{1}{2} \ln^2 \frac{Q}{Q_0}}$$

The final result for the “one emission out-of-gap” cross-section is

$$\sigma_{1,\mathrm{SLL}} = -\sigma_0 \left(\frac{2\alpha_s}{\pi}\right)^4 \boxed{\ln^5\left(\frac{Q}{Q_0}\right)} \pi^2 Y \frac{(3N^2-4)}{480}$$

<u>DGLAP evolution fails above the veto scale, whence the soft and collinear evolution can no longer be factorized – this is generic whenever real emission is not summed over inclusively.</u>

Not unrelated is the recent finding of del Duca, Duhr, Gardi, Magnea & White, that the gluon reggeization breaks down at NNLL. arXiv:1109.3581

$$M^{gg\to gg} = \left(\frac{s}{-t}\right)^{C_A K\left(\alpha_s(\mu^2),\varepsilon\right)} Z_1 H_t^{gg\to gg}$$

$$\widetilde{Z}\left(\frac{s}{t},\alpha_s(\mu^2),\varepsilon\right) = \left(\frac{s}{-t}\right)^{K\,\mathbf{T}_t^2} \exp\left\{\mathrm{i}\,\pi K\,\mathbf{T}_s^2\right\} \exp\left\{-\mathrm{i}\,\frac{\pi}{2}K^2\ln\left(\frac{s}{-t}\right)\left[\mathbf{T}_t^2,\mathbf{T}_s^2\right]\right\} \quad (3.8)$$
$$\times\exp\left\{\frac{K^3}{6}\left(-2\pi^2\ln\left(\frac{s}{-t}\right)\left[\mathbf{T}_s^2,[\mathbf{T}_t^2,\mathbf{T}_s^2]\right] + \mathrm{i}\pi\ln^2\left(\frac{s}{-t}\right)\left[\mathbf{T}_t^2,[\mathbf{T}_t^2,\mathbf{T}_s^2]\right]\right)\right\}\exp\left\{\mathcal{O}\left(K^4\right)\right\}.$$

- Generic to any observable which is not fully inclusive over real emissions, e.g. any exclusive n-jet cross-section.

- This can be thought of as the perturbative tail of the underlying event. Roughly speaking – any observable that is influenced by the UE will suffer. Even the single jet inclusive cross-section ought to be affected to some degree but not DIS observables (no Coulomb exchanges = no UE) and not Drell-Yan (full inclusive over real emissions).