A New Approach to String Cosmology

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Contents

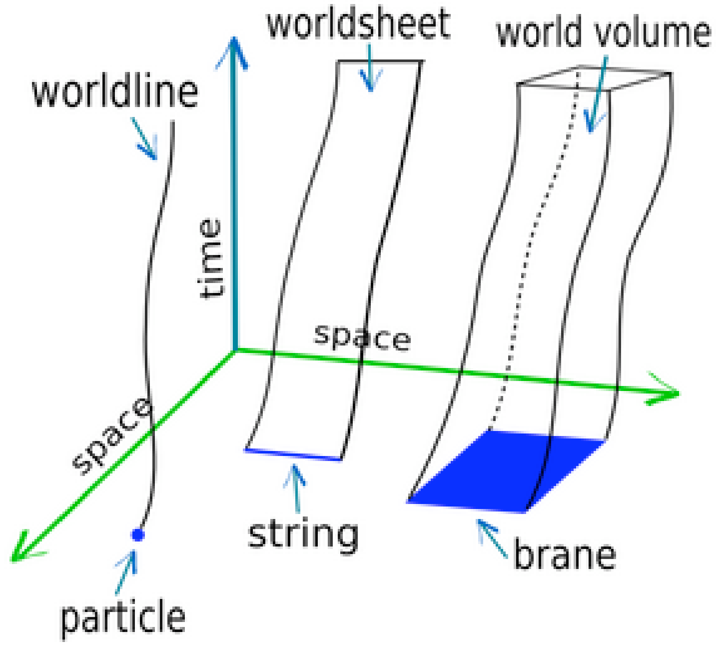
- Why String Cosmology?
- String Theory
- Conventional Approach to SC
- Our New Approach
- Cosmology Applications

Why String Cosmology

- Early Universe: need gravity + QFT united
- String scales (plank-scale) needed (e.g.Black Holes)
- Advances in string theory make predictions and “realistic” models feasible (e.g. inflation)
- Cosmology as a test of string theory (e.g cosmic strings)

String Theory

- General Relativity as a quantum field theory
- One-D String vs. Point as Fundamental
- High Energy Gravity Divergences “Smeared” over String Length



String Theory

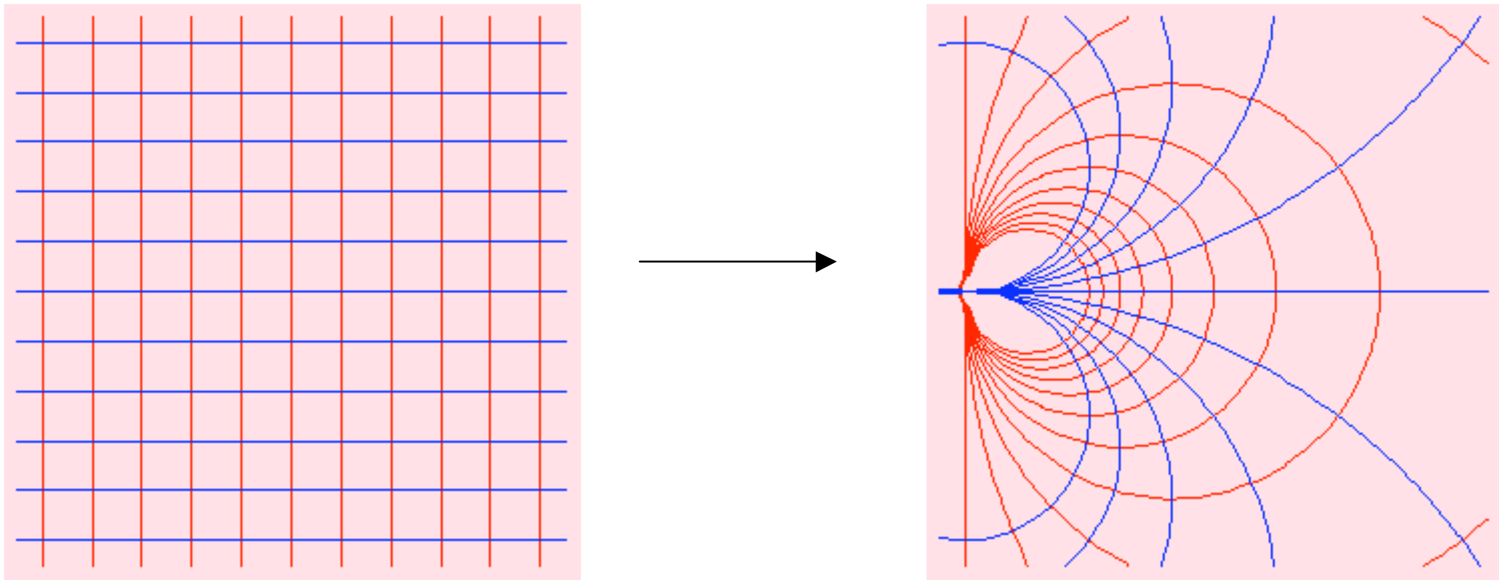
Polyakov action in flat spacetime

A 2-D Quantum Field Theory

$$S_P(X^\mu, \gamma) = -\frac{1}{4\pi\alpha'}\int_M d\tau d\sigma \sqrt{-\gamma}\left(\gamma^{\alpha\beta}\partial_\alpha X^\mu \partial_\beta X^\nu \eta_{\mu\nu} + \frac{\lambda}{4\pi}R\right)$$

$$\sigma^a = (\tau, \sigma)\ ,\ X^\mu = \left(X^0,\, X^1,\, \ldots X^{D-1}\right)$$

Conformal Invariance



$$\gamma_{\alpha\beta} \longrightarrow \exp(2\omega)\gamma_{\alpha\beta}$$

World sheet metric

Basis of String Cosmolgy

- Typical String Cosmology model with curved metric has background fields - massless degrees of freedom

$$S_\sigma = \frac{1}{4\pi\alpha'} \int_M d^2\sigma \sqrt{g}[(g^{\alpha\beta} G_{\mu\nu}(X) + i\epsilon^{\alpha\beta} B_{\mu\nu}(X)) \partial_\alpha X^\mu \partial_\beta X^\nu + \alpha' R\Phi(x)]$$

Regge Slope

(role of h-bar in QFT – sets length of string)

$g = det(g_{ab})$

curved worldsheet metric

Graviton field

Kalb-Ramond Antisymmetric tensor field

Kinetic term, with X as scalar fields in target space

Curvature scalar of worldsheet

Dilaton field

Beta Functions

Beta functions to one loop (first order in α')

$$\beta^{g(1)}_{\mu\nu} = R_{\mu\nu} + 2\nabla_\mu\nabla_\nu\phi - \frac{1}{4}H_{\mu\rho\sigma}H_\nu{}^{\rho\sigma}$$

$$\beta^{B(1)}_{\mu\nu} = -\frac{1}{2}\nabla^\rho H_{\rho\mu\nu} + \partial^\rho\phi H_{\rho\mu\nu}$$

$$\beta^{\phi(1)} = \boxed{\frac{D-26}{6\alpha'}} - \frac{1}{2}\nabla^2\phi + \partial^\rho\phi\partial_\rho\phi - \frac{1}{24}H_{\mu\nu\rho}H^{\mu\nu\rho}$$

Where:

$$H_{\rho\mu\nu} = \partial_\rho B_{\mu\nu} + \partial_\mu B_{\nu\rho} + \partial_\nu B_{\rho\mu}$$

Classical, flat space time,
free string beta function – hence D=26

Conventional String Cosmology

- Start with an action on the world sheet (a 2-D Field Theory)
- Calculate the one loop beta functions and set to zero
- Resulting constraints define the effective low energy space-time action and EoM of theory
- Generally only consider time variation
- This is a perturbative approach

New Approach - Step 1

- A non-perturbative solution to the bare action

$$S = \frac{1}{4\pi}\int d^2\xi\sqrt{\gamma}\left\{\lambda\left(\gamma^{ab}\eta_{\mu\nu} + \varepsilon^{ab}a_{\mu\nu}\right)\partial_a X^\mu \partial_b X^\nu + R^{(2)}\phi_{bare}(X^0)\right\}$$

Equation X

$$\begin{aligned}\dot{\Gamma}_\lambda &= \frac{1}{4\pi}\int d^2\xi\sqrt{\gamma}\left(\gamma^{ab}\eta_{\mu\nu} + \varepsilon^{ab}a_{\mu\nu}\right)\partial_a X^\mu \partial_b X^\nu \\ &\quad + \frac{1}{4\pi}\mathrm{Tr}\left\{\left(\gamma^{ab}\eta_{\mu\nu} + \varepsilon^{ab}a_{\mu\nu}\right)\frac{\partial}{\partial\xi^a}\frac{\partial}{\partial\zeta^b}\left(\frac{\delta^2\Gamma_\lambda}{\delta X^\mu(\zeta)\delta X^\nu(\xi)}\right)^{-1}\right\}\end{aligned}$$

New Approach - Step 2

- Assume form of some fields (dilaton, spatial invariance)
- Conformal invariance to 1st order:

$$\phi = \text{constant} + \phi_0 \ln X^0$$

$$H_{\rho\mu\nu}(X^0) \propto e^{\frac{m}{\phi_0}\phi(X^0)} \varepsilon_{\rho\mu\nu\sigma}(X^0) \partial^\sigma b,$$

$$g_{\mu\nu}(X^0) \;=\; \kappa_0 \, \text{diag}((X^0)^{-2}, -(X^0)^{-n}, -(X^0)^{-n}, -(X^0)^{2m})$$

New Approach - Step 3

- Demonstration that, for our chosen solution, the beta functions are consistent to all orders in powers of Xo (time).
- By field redefinitions we can make beta functions vanish to all orders - a non-perturbative, exact solution!!

Cosmology Applications

- No need for extra dimensions (we choose D=4) - a big deal in string theory!
- Solutions for flat, isotropic; or anisotropic universes can be modelled.

$$
\begin{aligned}
a_1(t) = a_2(t) &= a_0 \; t^{1+n/(2\phi_0)} \\
a_3(t) &= \tilde{a}_0 \; t^{1-m/\phi_0}
\end{aligned}
$$

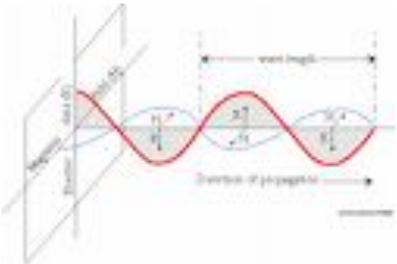
- Pseudo scalar, axion term comes out of string action - a candidate for dark matter

Cosmology Applications

- Cosmic Optics - we can couple the background fields to an EM term in the action

$$S_{eff} = \int d^4x\sqrt{-g}\left\{R - 2\partial_\mu\phi\partial^\mu\phi - \frac{e^{-4\phi}}{12}\tilde{H}_{\rho\mu\nu}\tilde{H}^{\rho\mu\nu} - \frac{e^{-2\phi}}{4}F_{\mu\nu}F^{\mu\nu}\right\}$$

Produces modified Maxwell's equations resulting in :



$$\mathbf{E} = \frac{\mathbf{E_0}}{t}\exp\left(-i\frac{\epsilon}{2}\ln t\right)\cos(t - x^3).$$

Damping Due to dilaton

Optical rotation

Motion in z-direction

Future Work

- Inflation configuration
- Generalisation to more than D=4
- A Supersymmetric model
- Potential links to dark matter, dark energy

References

Jean Alexandre, Nick Mavromatos, D Tanner, Kings College, London

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