Monte Carlo at NLO

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Young Theorists' Forum, 15-16th May 2009

Outline

Motivation

- LO accuracy is not enough
  - for many processes at the LHC the experimental precision will be such that at least NLO precision is required in predictive simulations
- Analytic NLO (and even NNLO) calculations have been performed for processes which will be important at the LHC
- However, analytic calculations are not sufficient
  - complicated phase space means all but simplest observables impossible to calculate analytically
  - want to combine NLO matrix element predictions consistently with parton shower and hadronisation to produce hadronic level predictions
- Therefore, NLO matrix elements must be implemented in Monte Carlo generators

The importance of neutral current Drell-Yan

- Parton distribution functions
  - low mass Z-production sensitive to PDFs at small x
- Detector calibration
  - comparison of M_Z and Γ_Z with LEP measurements
- Measurement of effective weak mixing angle
  - using forward-backward asymmetry near the Z pole
- Extraction of M_W
  - from the ratio of $W \to l\nu$ and $Z \to l^+l^-$ cross sections
- New physics searches
  - extra neutral gauge bosons, effects of large extra dimensions, composite quarks and leptons

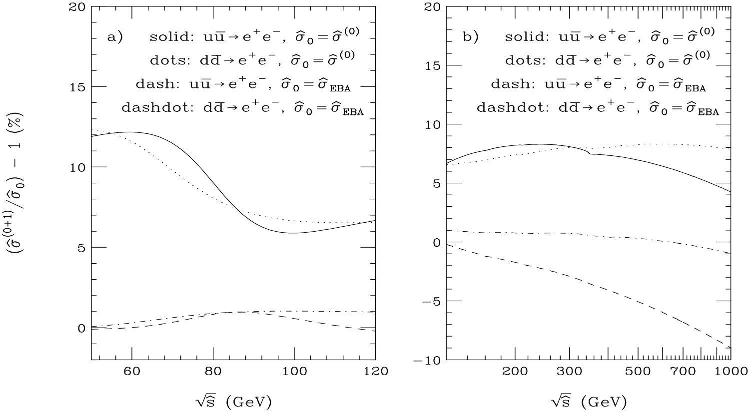

Figure: Relative corrections to total cross sections for $u\bar{u} \to e^+e^-$ and $d\bar{d} \to e^+e^-$

Baur et al. hep-ph/0108274

QCD NLO Cross Section

$$\sigma_{NLO} = \int_m d\sigma_{LO} + \int_m d\sigma_V + \int_{m+1} d\sigma_R$$

Difficulties in implementing NLO in Monte Carlo

- Calculation of virtual contribution
  - a lot of progress has been made here recently
  - general purpose automated 1-loop programs on the horizon
- Treatment of soft and collinear singularities
- Matching NLO matrix element with Parton Shower
  - POWHEG
  - MC@NLO

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Soft and collinear singularities

$$\sigma_{NLO} = \int_m d\sigma_{LO} + \int_m d\sigma_V + \int_{m+1} d\sigma_R$$

- Virtual and real contributions separately divergent
- Infinities cancel when combined
- Integrated over different phase spaces before combination

$\Rightarrow$ Completely unsuitable for numerical integration

Phase space slicing vs subtraction methods

Two main techniques to overcome this problem:

- Phase space slicing
  - introduce phase space cut δ
  - if δ is small enough, result should be independent of it
  - a rather crude, and increasingly unpopular method
- Subtraction methods

Subtraction Term

A subtraction term is introduced to cancel divergences:

$$\int_m \mathrm{d}\sigma_V + \int_{m+1} \mathrm{d}\sigma_R \quad = \quad \int_m \left[\mathrm{d}\sigma_V + \int_1 \mathrm{d}\sigma_A\right] + \int_{m+1} [\mathrm{d}\sigma_R - \mathrm{d}\sigma_A] \ .$$

This simple step solves the numerical problems, provided that the subtraction term

- exactly matches the pointwise singular behaviour of the real matrix element,
- contains no other divergences and is otherwise convenient for Monte Carlo integration, and
- is of a form such that it is possible to integrate it analytically over the one-particle phase space in d-dimensions.

Sounds perfect... but is it possible to find such a term?

Process-dependent subtraction terms

- Introduced by Ellis, Ross and Terrano in 1981 for the case of $e^+ + e^- \to 3$ jets
- Subsequently applied to other processes, but method must be adapted to each process

Nucl.Phys.B178:421,1981

Catani-Seymour Dipole Subtraction

- Completely general version of the subtraction method.
- Set of universal counter terms which can be used for *any* NLO QCD process.
- Relies on the factorisation of the soft and collinear divergences.

hep-ph/9605323

Factorisation of soft and collinear divergences

- Consider an m-parton leading order matrix element
- Choose two external partons and label them i and k
- Consider parton i emitting a soft or collinear parton labeled j
- Reshuffle the momenta of the three partons to make all three on-shell

The resulting matrix element can be written symbolically as

$$|\mathcal{M}_{m+1}|^2 \quad \rightarrow \quad |\mathcal{M}_m|^2 \otimes \mathbf{V}_{ij,k} \ ,$$

where $\mathbf{V}_{ij,k}$ is a singular factor, and depends only on the quantum numbers of i,j and k, and on their momenta. The singular factor is otherwise completely process independent.

Constructing the subtraction term

- This approximation of the matrix element is valid in the limit of
  - j soft, or
  - i and j collinear.
- To construct subtraction term, sum over all possible dipole configurations.
- Subtraction term therefore contains all soft and collinear divergences present in the real emission matrix element.

$$\sum_{k \neq i,j} \mathcal{D}_{ij,k} \quad = \quad \sum_{k \neq i,j} \frac{1}{p_i . p_j} \langle \mathcal{M}_m^{ij,k} | \frac{\mathbf{T}_k . \mathbf{T}_{ij}}{\mathbf{T}_{ij}^2} \mathbf{V}_{ij,k} | \mathcal{M}_m^{ij,k} \rangle$$

One-particle phase space integration

- Unique mapping of momenta back to leading order configuration for each dipole term.
  - $p_i, p_j, p_k \to \tilde{p}_{ij}, \tilde{p}_k$
- Phase space of the extra emitted parton can be factorised.
- Splitting function $\mathbf{V}_{ij,k}$ has been analytically integrated over this single particle phase space in *d*-dimensions.
- This results in a term which has the simple structure

$$\int d\Phi_1 \mathcal{D}_{ij,k} = \mathcal{V}_{ij,k} \langle \mathcal{M}_m^{ij,k} | \frac{\mathbf{T}_k . \mathbf{T}_{ij}}{\mathbf{T}_{ij}^2} | \mathcal{M}_m^{ij,k} \rangle .$$

- Summing over these integrated subtraction terms gives the counter term for the divergent virtual corrections.

Full cross section

$$\begin{aligned}\sigma_{NLO} &= \int_m \mathrm{d}\sigma_{LO} \\ &+ \int_m \left[\mathrm{d}\sigma_V + \mathrm{d}\Phi_m \sum_{k\neq i,j} \mathcal{V}_{ij,k} \langle \mathcal{M}_m^{ij,k} | \frac{\mathbf{T}_k . \mathbf{T}_{ij}}{\mathbf{T}_{ij}^2} | \mathcal{M}_m^{ij,k} \rangle \right] \\ &+ \int_{m+1} \left[\mathrm{d}\sigma_R - \mathrm{d}\Phi_{m+1} \sum_{k\neq i,j} \mathcal{D}_{ij,k} \right] .\end{aligned}$$

Summary

- NLO precision is required for LHC physics.
- NLO matrix elements are much trickier to implement in a Monte Carlo event generator than their LO counterparts.
- Catani-Seymour dipole subtraction can be used to make divergent NLO contributions suitable for Monte Carlo integration.